$W_A = \sum_{A'} w_{(A',A)}$ and $W_I = 1$ [A′ is a predecessor of A]

2. The backward pass: identical to the single-pass algorithm with the exception that the weights $w_{(A,B)}$ are used to work out the split of trips rather than the splitting fators F_i.

Example 10.3: A practical problem with Dial's assignment is that it is biased against trunk routes as opposed to secondary links. Consider the problem of a town served by a bypass and a town-centre route with three small variations as illustrated in Figure 10.6. Assume also that there are 4000 trips from A to B and that all routes have approximately the same cost.

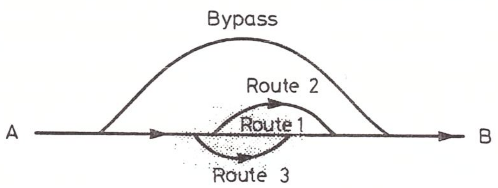

Figure 10.6 Town served by a bypass and three city-centre routes

In this case Dial's algorithm would split the 4000 trips as follows: 1000 via the bypass and 1000 via each of the town-centre routes. However, most users would regard this problem as one with only two alternatives: bypass or town centre. Recall the discussion about the independence of irrelevant alternatives property of the logit model in Chapter 5. Dial's runs into trouble when it considers every possible route even if some permutations or combinations of links may differ just in a few percentage points of their total cost. In behavioural terms Dial ignores the correlation between similar routes. In practice, Dial tends to allocate more traffic to dense sections of the network with short links, compared with sparser parts of the network with relatively longer links. In fact, coding strategies for networks can affect the allocation of flows.

10.5 CONGESTED ASSIGNMENT

10.5.1 Wardrop's equilibrium

If one ignores stochastic effects and concentrates on capacity restraint as a generator of a spread of trips on a network, one should consider a different set of models. For a start, capacity restraint models have to make use of functions relating flow to the cost (time) of travel on a link. These models usually attempt, with different degrees of success, to approximate to the equilibrium conditions as formally enunciated by Wardrop 1952):

> *Under equilibrium conditions traffic arranges itself in congested networks in such a way that no individual trip maker can reduce his path costs by switching routes.*

If all trip makers perceive costs in the same way (no stochastic effects):

> *Under equilibrium conditions traffic arranges itself in congested networks such that all used routes between an O–D pair have equal and minimum costs while all unused routes have greater or equal costs.*

This is usually referred to as Wardrop's first principle, or simply Wardrop's equilibrium. It is easy to see that if these conditions did not hold, at least some drivers would be able to reduce their costs by switching to other routes.

Example 10.4: Consider again the case of a bypass and a single town-centre route as discussed in section 10.2.2 (Figure 10.2). Assume now that the absolute capacity restriction for each route is replaced with two corresponding time–flow relationships as illustrated in Figure 10.7.

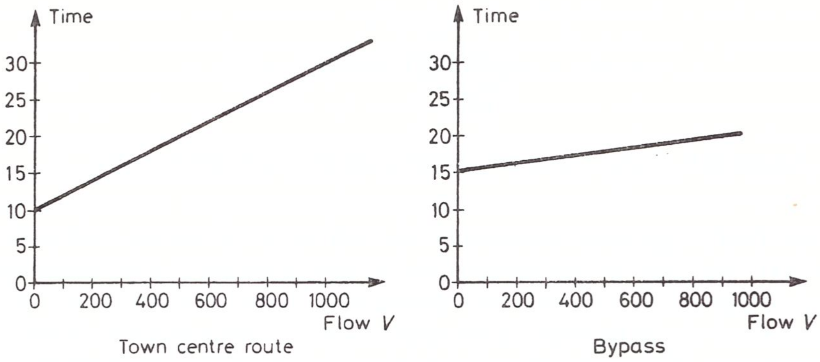

Figure 10.7 Time-flow relationships for Figure 10.2

The flows on the two routes will satisfy Wardrop's equilibrium when the corresponding costs are identical. In this case it is relatively simple to write two equations for travel time versus flow and equate them to find the equilibrium solution, for example:

$$C_b = 15 + 0.005V_b \tag{10.14a}$$

$$C_t = 10 + 0.02V_t \tag{10.14b}$$

where C_b and C_t are travel costs via the bypass and the town-centre routes respectively, and V_b and V_t are their corresponding flows.

By equating C_b to C_t it is possible to find, in this simple case, the direct solution to Wardrop's equilibrium as a function of the total flow $V_b + V_t = V$:

$$15 + 0.005V_b = 10 + 0.02(V - V_b)$$

that is:

$$V_b = 0.8V - 200 \tag{10.15}$$

Expression (10.15) has meaning only for non-negative flows, i.e. for V greater than or equal to $200/0.8 = 250$. For $V < 250$, $C_t < C_b$, $V_b = 0$ and $V_t = V$, i.e. all traffic chooses the town-centre route. For situations where $V > 250$ the two routes will be used; for example, the reader can verify that for $V = 2000$ the equilibrium flows are $V_b = 1400$ and $V_t = 600$ and the costs by each route are 22 minutes.

The same idea would apply to flows on networks where the costs of travel by each of the routes used between two points will be the same under Wardrop's equilibrium. The problem is, of course, that in anything but the simplest cases it is not possible to solve the equilibrium flows algebraically; rather an algorithmic solution method is required.

Several techniques have been proposed as reasonable approximations to Wardrop's equilibrium: some of them are simple heuristic approaches and the most interesting ones follow a more rigorous mathematical programming framework. In order to compare these algorithms against each other the following properties are of interest:

- Is the solution stable?
- Does it converge to the correct solution (Wardrop's equilibrium)?
- Is it efficient in terms of computational requirements?

The indicator δ, defined in the following equation, is often used to measure how close a solution is to Wardrop's equilibrium:

$$\delta = \frac{\sum_{ijr} T_{ijr}(C_{ijr} - C_{ij}^{*})}{\sum_{ij} T_{ij} C_{ij}^{*}} \tag{10.16}$$

where $C_{ijr} - C_{ij}^{*}$ is the excess cost of travel over a particular route relative to the minimum cost of travel for that (i, j) pair. These costs are calculated after the last iteration has been performed and total flows obtained for each link. Therefore δ is a measure of the total cost of excess travel via less than optimal routes, with denominator introduced so that the measure is recorded in relative rather than absolute terms.

Wardrop (1952) proposed an alternative way of assigning traffic onto a network and this is usually referred to as his second principle:

> *Under social equilibrium conditions traffic should be arranged in congested networks in such a way that the average (or total) travel cost is minimised.*

This is a *design* principle, in contrast with his first principle which endeavours to model the behaviour of individual drivers trying to minimise their own trip costs. The second principle is oriented towards transport planners and engineers trying to manage traffic to minimise travel costs and therefore achieve an optimum *social equilibrium*. In general the flows resulting from the two principles are not the same but one can only expect, in practice, traffic to arrange itself following an approximation to Wardrop's first principle, i.e. *selfish* or *users' equilibrium*.